**UNIVERSITY OF TORONTO  
Faculty of Arts and Science**

**April 2017 EXAMINATIONS**

**PHL245H1-S**

**Alex Koo**

**Duration - 3 hours**

**No Aids Allowed**

Last Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

First Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Student Number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Answer **ALL** questions on the exam paper.

Use examination booklets for rough work if needed.

If you need further space, use an examination booklet and clearly indicate on the exam paper where your solution is.

The exam consists of 16 pages. Pages 2-14 have questions on them.

The final two pages (15-16) are blank lines for use if needed.

Part I: Semantics (30 marks)

1. When we do a conditional derivation, we get to assume the antecedent in order to show the consequent. With reference to the truth-table of the conditional, explain why this assumption makes sense. (3)

2. Provide an intensional interpretation that shows the following argument is invalid. (3)

∀x(Gx↔~Fx). ∀x∃y(Gx→Fy∧M(xy)). ∴ ~∃y(Fy∧∀x(Fx→M(xy))).

3. Provide an English explanation that demonstrates why the following set of sentences is inconsistent. (4)

{∃x∀y(Fx∧~D(xy)), ∀y(Fy→∃xD(yx))}

4. Provide a finite extensional interpretation/model that demonstrates the following sentence is not a tautology/logical truth. (4)

∀x(Fx↔Gx)∨(Fa∧∃x(Fx∧Gx∧~H(xx))→~∃x(Gx∧∀y(Fy∧H(yx))))

5. a) Provide a truth-functional expansion of the following set of sentences using a  
 universe of discourse with two members. (4)

{~∀x(D(xx)→~Ax), ∃x∀y(D(xy)∧Gx), ∀x(Ax→∃y~D(xy))}

b) Provide a finite extensional interpretation/model that demonstrates the set of sentences is consistent. (1)

6. What is the difference between the definition of validity in **SENTENTIAL** logic versus in **PREDICATE** logic? Use this difference to explain why we need to use models/interpretations in predicate logic semantics. (3)

7. Provide a shortened truth-table that demonstrates the following argument is invalid. (3)

(W↔Q)→R∨Q. P∧~W. ∴ P↔Q.

8. In three-valued logic, there are three possible truth-values: True (T), False (F), and Unknown (U). Below are the completed three-valued truth-tables for Negation, Conjunction, and Disjunction, as well as an empty table for the Conditional.

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| **P** | **Q** | ~P | P∧Q | P∨Q | P→Q |
| T | T | F | T | T |  |
| T | U | F | U | T |  |
| T | F | F | F | T |  |
| U | T | U | U | T |  |
| U | U | U | U | U |  |
| U | F | U | F | U |  |
| F | T | T | F | T |  |
| F | U | T | F | U |  |
| F | F | T | F | F |  |

1. One way to understand the Conditional P→Q in three-valued logic is by making it equivalent to ~P∨Q. Fill in the P→Q column above using this understanding. (2)
2. Given that P is True (T) and Q is Unknown (U), circle the truth-value of the following sentence. (1)

Q∨(~Q∧(Q∨P))

TRUE UNKNOWN FALSE

c) Give one reason for why we would want to use a three-valued logical system.   
 Briefly justify this reason. (2)

Part II: Symbolization (36 Marks)

Symbolize questions 1-8, and translate question 9 using the provided abbreviation schemes. Read the instructions for question 10 carefully.

1. Unless coffee and cigarettes are good for you, they will be neither cheap nor socially acceptable. (3)

A1: *a* is cheap. C1: *a* is coffee. D1: *a* is socially acceptable. F1: *a* is a cigarette.

G1: *a* is good for you.

2. Not everyone rides bicycles. (3)

A1:*a* is a person. B1: *a* is a bicycle. D2: *a* rides *b*.

3. Only if Steven’s mother is an engineer will she (Steven’s mother) go to graduate school. (4)

a0: Steven. a1: The mother of *a*. E1: *a* is an engineer. G1: *a* is a graduate school.   
D2: *a* will go to *b*.

4. Jenny is the coolest farmer in Dallas exactly on the condition that she doesn’t like kale. (4)

b0: Jenny. d0: Dallas. F1: *a* is a farmer. M1: *a* is kale. C2: *a* is cooler than *b*.  
L2: *a* likes *b*. N2: *a* is in *b*.

5. Everyone except for Jodie’s daughter, who is Richard’s son’s wife, drinks water. (4)

a0: Jodie. b0: Richard. b1: The son of *a*. d1: The daughter of *a*. g1: The wife of *a*.   
D1: *a* is a person. H1: *a* is water. D2: *a* drinks *b*.

6. Some generic cat is always clawing some specific dog. (4)

C1: *a* is a cat. D1: *a* is a dog. K1: *a* is a time. C3: *a* is clawing *b* at time *c*.

7. There is only one cheese that Sarah likes, but it’s expensive. (4)

b0: Sarah. C1: *a* is a cheese. E1: *a* is expensive. L2: *a* likes *b*.

8. Amongst athletes, being smart is necessary for being good; and in that case they’ll get on a team. (4)

A1: *a* is an athlete. B1: *a* will get on a team. G1: *a* is good. H1: *a* is smart.

9. Translate the following symbolic sentence into an IDIOMATIC English sentence using the abbreviation scheme provided. (3)

∀x(Dx∧∃y∃z(Cy∧Cz∧y≠z∧H(xy)∧H(xz))→∀y(Dy∧∃w∃z(Cw∧Cz∧w≠z∧H(yw)∧H(yz))→x=y))

C1: *a* is a crime. D1: *a* is a person. H2: *a* saw *b*.

10. Define a new operator in our system **℩** (called ‘cane’). This operator combines with a variable, and together with a predicate we get a formula. For example, ℩xFx is a formula. We can understand ℩x as saying ‘the thing such that’ – ℩x is the definite descriptor. If we define F to mean F1: *a* is on my desk, then ℩xFx means ‘*the thing* on my desk’. ℩xφx, where φx is a formula, thus picks out a **term** or **specific individual**, and can be used as a term in our symbolizing.

**Using the operator ℩,** symbolize the following sentence. (3)

The cup on my desk is owned by Harry’s mom.

a1: The mom of *a*. h0: Harry. C1: *a* is a cup. F1: *a* is on my desk. O2: *a* owns *b*.

Part III: Derivations (34 marks)

1. Show the following argument is valid using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (8)

(P→~∀zFz)→∃x∀yB(xy). ∀w∃z~B(wz). ∴ ∀x∀y(Fx∨~G(yx))

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2. Show the following statement is a theorem of logic using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (8)

∴ (∀x∀yG(yx)∧(G(ab)→∃xF(a(x)a)))→∃x(F(xa)∧∃yG(b(y)x))

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3. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (9)

∃z∀xM(xb(z))∨∀x∀yD(xyb(xy)). ∃x~∃zD(xa(x)z). ∴ ∃yM(a(y)y)

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4. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (9)

~(∀xF(xx)→∃z(Bz∧Gz)). ∴ ∃xF(xa(b))↔~∀x∃y(Gx∧By)

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Total = 100 Marks.

Extra Lines. If you use these, clearly indicate how the grader should read your proof.

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Extra Lines. If you use these, clearly indicate how the grader should read your proof.

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